RADIATIVE HEAT TRANSFER IN A GAS-FILLED SPHERICAL RING

A. S. Nevskii

UDC 536.3

An analysis is made here of the radiative heat transfer in the gas-filled space between two concentric spheres.

A completely correct solution to the problem of radiative heat transfer between a gray isothermal medium and isotropically reflecting boundary surfaces is, at the present time, known only for the simplest geometrical shapes: an infinitely long stratum, a sphere, and an infinitely long cylinder. In the case of a stratum or a sphere with mirror surfaces such a solution has been obtained for a gray medium as well as for a selectively emitting gas [1].

We will consider here the radiative heat transfer between an isothermal gas, gray or selective, occupying the space of a spherical ring bounded by an inner surface k and an outer surface i which are either mirror or isotropically reflecting.

Radiative Heat Transfer in the Case of a Gray Gas and Isotropically Reflecting Surfaces. The absorptivities of the medium will be equal here to its emissivities. The equations of heat balance for each surface are:

$$Q_{\mathrm{inc}\,i} = F_i a_{\mathrm{G}i} \sigma_0 T_{\mathrm{G}}^4 + Q_{\mathrm{eff}\,i} \varphi_{ii} (1 - a_{ii}) + Q_{\mathrm{eff}} (1 - a_{ik}), \qquad (1)$$

$$Q_{\mathrm{inc}\,k} = F_k a_{ik} \sigma_0 T_{\mathrm{G}}^4 + Q_{\mathrm{eff}}\, \varphi_{ik} (1 - a_{ik}). \qquad (2)$$

The magnitudes of thermal fluxes Q_{inc} and Q_{eff} are related according to the equation in [1, Chapter 1]:

$$Q_{\mathrm{eff}} = Q_{\mathrm{inc}} R + F A \sigma_0 T^4. \qquad (3)$$

The gas has an absorptivity referred to radiation from surface i

$$a_{G/i} = \omega a_{ik} + (1 - \omega)\, a_{ii}. \qquad (4)$$

The angular coefficients are

$$\varphi_{ik} = \omega, \quad \varphi_{ii} = 1 - \omega. \qquad (5)$$

After solving these equations, we easily find the resultant amount of heat transfer at surfaces i and k. Omitting all the intermediate calculations, which are unwieldy but not difficult in principle, we show the final result:

$$Q_{\mathrm{Ri}} = F_i A_i \sigma_0 \{[a_{G/i} + R_k \varphi_{ik} a_{ik} (1 - a_{ik})](T_{\mathrm{G}}^4 - T_i^4) + A_k \varphi_{ik} (1 - a_{ik}) (T_k^4 - T_i^4)\} \{1 - R_i \varphi_{ii} (1 - a_{ii}) - R_i R_k \varphi_{ik} (1 - a_{ik})^2\}^{-1}, \qquad (6)$$

$$Q_{\mathrm{Rk}} = F_k A_k \sigma_0 \{[a_{ik} + R_i a_{G/i} (1 - a_{ik}) - R_i \varphi_{ii} a_{ik} (1 - a_{ii})] (T_{\mathrm{G}}^4 - T_k^4) + A_i (1 - a_{ik}) (T_i^k - T_k^k)\} \{1 - R_i \varphi_{ii} (1 - a_{ii}) - R_i R_k \varphi_{ik} (1 - a_{ik})^2\}^{-1}, \qquad (7)$$

$$Q_{\mathrm{RG}} = -(Q_{\mathrm{Ri}} + Q_{\mathrm{Rk}}). \qquad (8)$$

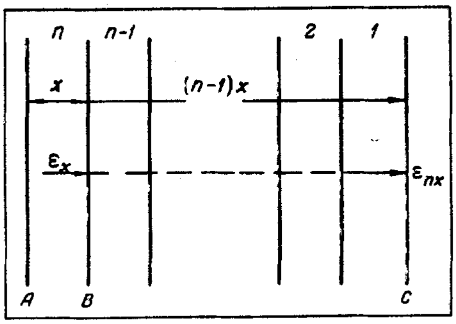

Fig. 1. Schematic diagram for formulas (11)-(13).

Radiative Heat Transfer with Mirror Reflection at the Surfaces. The resultant heat transfer at surface k is

All-Union Scientific-Research Institute of Metallurgical Thermoengineering, Sverdlovsk. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 24, No. 3, pp. 400-406, March, 1973. Original article submitted April 3, 1972.

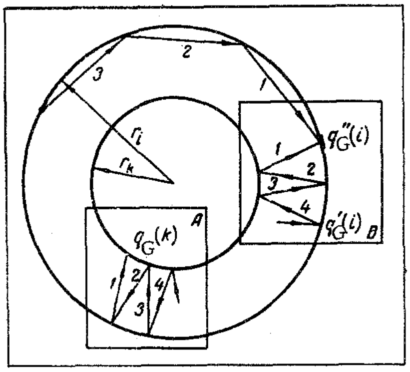

Fig. 2

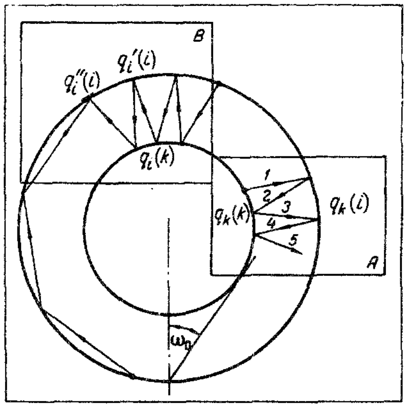

Fig. 3

Fig. 2. Schematic diagram of radiant fluxes originating in the gas.

Fig. 3. Schematic diagram of radiant fluxes originating at the surfaces.

$$Q_{Rk} = F_k \left[q_G(k) + \frac{1}{\omega} q_i(k) + q_k(k) - q_c(k) \right], \tag{9}$$

and at surface i is

$$Q_{Ri} = F_i \left[q'_G(i) + q''_G(i) + q'_i(i) + q''_i(i) + \omega q_k(i) - q_c(i) \right]. \tag{10}$$

We will first derive a few auxiliary relations. We consider a beam of rays in the isothermal gas along path x (Fig. 1). These rays traverse in the gas n−1 segments of length x each. The transmissivity of the medium here along the path (n−1)x

$$D_{(n-1)x} = d_1 d_2 \cdots d_{n-1}, \tag{11}$$

according to the definition, with d_1, d_2, . . . denoting the transmissivities of the medium in each segment, will be found with the aid of formula (2-136) in [1]:

$$D_{(n-1)x} = \frac{\varepsilon_{nx} - \varepsilon_{(n-1)x}}{\varepsilon_x}. \tag{12}$$

Considering the surface radiation absorbed by the gas, we have

$$D_{(n-1)x} = 1 - a_{(n-1)x}, \tag{13}$$

with $a_{(n-1)x}$ denoting the absorptivity of the gas with respect to this radiation.

These relations apply also to rays passing through a gas layer of thickness x along a broken-line path as a result of mirror reflections at the boundary surfaces.

The total gas radiation incident on a surface element dF_k (Fig. 2A) or dF_i (Fig. 2B) in any direction whatever consists of direct radiation from the gas along path 1 and of radiation coming from it along such segments as 2, 3, . . . reaching surface k or i after a series of reflections at these surfaces and absorption by the gas. The amount of energy absorbed by surfaces k and i can be calculated by adding the sequence of rays and then integrating over all angles. The result will be

$$q_G(k) = \frac{A_k \sigma_0 T_G^4}{\pi} \int_{2\pi} \varepsilon_x (1 + R_i d_1 + R_i R_k d_1 d_2 + \ldots) \cos\varphi \, d\omega, \tag{14}$$

$$q'_G(i) = \frac{A_i \sigma_0 T_G^4}{\pi} \int_{\omega_0} \varepsilon_x (1 + R_k d_1 + R_k R_i d_1 d_2 + \ldots) \cos\varphi \, d\omega. \tag{15}$$

The d_i products will be now replaced according to formula (11), after which we use expression (12):

$$q_G(k) = \frac{A_k \sigma_0 T_G^4}{\pi} \int_{2\pi} \left[\varepsilon_x + R_i (\varepsilon_{2x} - \varepsilon_x) + R_i R_k (\varepsilon_{3x} - \varepsilon_{2x}) + \ldots \right] \cos\varphi \, d\omega, \tag{16}$$

$$q_G'(i) = \frac{A_i\sigma_0 T_G^4}{\pi} \int\limits_{\omega_c} \left[\varepsilon_x + R_k(\varepsilon_{2x} - \varepsilon_x) + R_k R_i(\varepsilon_{3x} - \varepsilon_{2x}) + \ldots\right] \cos\varphi d\omega. \quad (17)$$

In order to transform Eqs. (16) and (17), we use the identity

$$\varepsilon_{kx} + R\left[\varepsilon_{(k+1)x} - \varepsilon_{kx}\right] = A\varepsilon_{kx} + R\varepsilon_{(k+1)x}, \quad (18)$$

after which we integrate the obtained expressions:

$$q_G(k) = \sigma_0 T_G^4 \left[A_i A_k \chi_1(i,\ k,\ T_G) + A_k^2 R_i \chi_2(i,\ k,\ T_G)\right], \quad (19)$$

$$q_G'(i) = \sigma_0 T_G^4 \varphi_{ik} \left[A_i A_k \chi_1(i,\ k,\ T_G) + A_i^2 R_k \chi_2(i,\ k,\ T_G)\right], \quad (20)$$

where

$$\begin{aligned} \chi_1(i,\ k,\ T_G) &= \varepsilon_r(i,\ k) + R_i R_k \varepsilon_{3r}(i,\ k) + \ldots, \\ \chi_2(i,\ k,\ T_G) &= \varepsilon_{2r}(i,\ k) + R_i R_k \varepsilon_{4r}(i,\ k) + \ldots \end{aligned} \quad (21)$$

Coefficients ε_{kr} in (21) represent the mean emissivities of the medium between surfaces i and k. The subscripts r, 2r, 3r, ... indicate that the respective values of ε refer to the original system with radii r_i and r_k, to a system of twice the dimensions, to a system of three times the dimensions, etc. respectively.

In addition to radiation multiply reflected between surfaces i and k, surface i receives also radiation emitted by the gas in directions beyond the other surface k (Fig. 2B). The amount of this radiation is determined in the same manner as $q_r'(i)$. We have the following expression for the energy of this gas radiation absorbed by surface i:

$$q_G''(i) = \sigma_0 T_G^4 A_i^2 \varphi_{ii} \chi_3(i,\ i,\ T_G), \quad (22)$$

where

$$\chi_3(i,\ i) = \chi_1(i,\ i) + R_i \chi_2(i,\ i) = \varepsilon_r(i,\ i) + R_i \varepsilon_{2r}(i,\ i) + R_i^2 \varepsilon_{3r}(i,\ i) + \ldots \quad (23)$$

In order to determine $q_k(i)$, $q_k(k)$, $q_i(k)$, $q_i'(k)$, and $q_i''(k)$, we examine the path which the rays emitted by each of the surfaces k and i traverse as a result of reflection at the surfaces, absorption by the surfaces, and absorption by the gas.

A unit area of surface k emits a radiant flux $A_k\sigma_0 T_k^4$ (Fig. 3A). One part of this flux is absorbed by surface i:

$$q_k(i) = \frac{A_k\sigma_0 T_k^4}{\pi} A_i \int\limits_{2\pi} (d_1 + R_1 R_k d_1 d_2 d_3 + \ldots) \cos\varphi d\omega, \quad (24)$$

and another part by the same surface k:

$$q_k(k) = \frac{A_k^2\sigma_0 T_k^4}{\pi} R_i \int\limits_{2\pi} (d_1 d_2 + R_i R_k d_1 d_2 d_3 d_4 + \ldots) \cos\varphi d\omega. \quad (25)$$

The d_i products in Eqs. (24) and (25) will now be replaced according to (11) and (13), the infinite series inside the brackets will then be summed, and the final expressions integrated

$$q_k(i) = A_i A_k \sigma_0 T_k^4 \left[\frac{1}{1 - R_i R_k} - \chi_{1a}(i,\ k,\ T_k,\ T_G)\right], \quad (26)$$

$$q_k(k) = A_k^2 R_i \sigma_0 T_k^4 \left[\frac{1}{1 - R_1 R_k} - \chi_{2a}(i,\ k,\ T_k,\ T_G), \quad (27)$$

with χ_{1a} and χ_{2a} denoting functions analogous to χ_1 and χ_2 in (21) but of black (gray) radiation absorptivities rather than of gas emissivities, T_k denoting the surface temperature, and T_G denoting the gas temperature to which they are referred.

In an analogous manner we derive expressions for the radiant fluxes emitted by surface i:

$$q_i(k) = A_i A_k \varphi_{ik} \sigma_0 T_i^4 \left[\frac{1}{1 - R_i R_k} - \chi_{1a}(i,\ k,\ T_i,\ T_G), \quad (28)$$

$$q_i'(i) = A_i^2 R_k \varphi_{ik} \sigma_0 T_i^4 \left[\frac{1}{1 - R_i R_k} - \chi_{2a}(i,\ k,\ T_i,\ T_G), \quad (29)$$

$$q_i''(i) = A_i^2\varphi_{ii}\sigma_0 T_i^4\left[\frac{1}{1-R_i} - \chi_{3a}(i,\ i,\ T_i,\ T_G)\right]; \tag{30}$$

and with the aid of these as well as formulas (9), (10), (19), (20), (22), (28), (29), and (30) we find the resultant amount of radiative heat transfer between surfaces k and i:

$$\frac{Q_{Rk}}{F_k} = \sigma_0\left\{T_G^4[A_iA_k\chi_1(i,\ k,\ T_G) + A_k^2R_i\chi_2(i,\ k,\ T_G)] - T_k^4[A_iA_k\chi_{1a}(i,\ k,\ T_k,\ T_G) + A_k^2R_i\chi_{2a}(i,\ k,\ T_k,\ T_G)]\right\} - \sigma_0A_iA_k[T_i^4\chi_{1a}(i,\ k,\ T_i,\ T_G) - T_k^4\chi_{1a}(i,\ k,\ T_k,\ T_G)] + \frac{A_iA_k\sigma_0(T_i^4 - T_k^4)}{1-R_iR_k}; \tag{31}$$

$$\frac{Q_{Ri}}{F_i} = \omega\sigma_0\left\{T_G^4[A_iA_k\chi_1(i,\ k,\ T_G) + A_i^2R_k\chi_2(i,\ k,\ T_G)] - T_i^4[A_iA_k\chi_{1a}(i,\ k,\ T_i,\ T_G) + A_i^2R_k\chi_{2a}(i,\ k,\ T_i,\ T_G)]\right\} + (1-\omega)\sigma_0A_i^2[T_G^4\chi_3(i,\ i,\ T_G) - T_i^4\chi_{3a}(i,\ i,\ T_i,\ T_G)] - \omega\sigma_0A_iA_k[T_k^4\chi_{1a}(i,\ k,\ T_k,\ T_G) - T_i^4\chi_{1a}(i,\ k,\ T_i,\ T_G)] + \frac{\omega A_iA_k\sigma_0}{1-R_iR_k}(T_k^4 - T_i^4). \tag{32}$$

Here Q_{RG} is found according to formula (8).

In the case of a gray medium, $\chi_{1a} = \chi_1$ and $\chi_{2a} = \chi_2$, both becoming independent of the temperature:

$$\frac{Q_{Rk}}{F_k} = \sigma_0[A_iA_k\chi_1(i,\ k) + A_k^2R_i\chi_2(i,\ k)](T_G^4 - T_k^4) + A_iA_k\sigma_0\chi_1(i,\ k)(T_i^4 - T_k^4) + \sigma_0\frac{A_iA_k(T_i^4 - T_k^4)}{1-R_iR_k}; \tag{33}$$

$$\frac{Q_{Ri}}{F_i} = \omega\sigma_0[A_iA_k\chi_1(i,\ k) + A_i^2R_k\chi_2(i,\ k)](T_G^4 - T_i^4) + (1-\omega)\sigma_0A_i^2\chi_3(i,\ i)(T_G^4 - T_i^4) - \omega\sigma_0A_iA_k\chi_1(i,\ k)(T_k^4 - T_i^4) + \frac{\omega A_iA_k\sigma_0}{1-R_iR_k}(T_k^4 - T_i^4). \tag{34}$$

When gases are selective emitters, then the absorptivities of the medium are not equal to its emissivities. For gaseous carbon dioxide and water vapor, for example, the absorptivities referred to radiation from black or gray walls have been determined in [2, 3]. For a unidirectional radiation we have

$$a(x) = \left(\frac{T_G}{T_s}\right)^n \varepsilon\left(x\frac{T_s}{T_G},\ T_s\right), \tag{35}$$

with n = 0.65 for carbon dioxide and n = 0.45 for water vapor. We will now rewrite expression (35) for each ray between any surfaces F_i and F_k whatever. We then multiply it by $\cos\vartheta_i \cos\vartheta_k dF_i dF_k/F_i\pi x^2$ and integrate, obtaining on the left-hand side the absorptivity of the medium between the two surfaces times the angular coefficient for radiation from surface i to surface k, and on the right-hand side the quantity $(T_G/T_s)^n\varphi_{ik}\varepsilon(ST_s/T_G,\ T_s)$, where S denotes the characteristic geometrical dimension of the radiation system. Therefore,

$$a(s) = \left(\frac{T_G}{T_s}\right)^n \varepsilon\left(s\frac{T_s}{T_G},\ T_s\right).$$

Formula (36) can be used for expressing χ_{1a} and χ_{2a} for the gas-filled space in terms of the following equalities:

$$\chi_{1a}(i,\ k,\ T_s,\ T_G,\ r) = \left(\frac{T_G}{T_s}\right)^n \chi_1\left(i,\ k,\ T_G,\ r\frac{T_s}{T_G}\right),$$

$$\chi_{2a}(i,\ k,\ T_s,\ T_G,\ r) = \left(\frac{T_G}{T_s}\right)^n \chi_2\left(i,\ k,\ T_G,\ r\frac{T_s}{T_G}\right),$$

$$\chi_{3a}(i,\ i,\ T_s,\ T_G,\ r) = \left(\frac{T_G}{T_s}\right)^n \chi_3\left(i,\ i,\ T_G,\ r\frac{T_s}{T_G}\right).$$

With the aid of these solutions, one can then determine the radiative heat transfer within a spherical layer of a gray medium with isotropic reflection from the boundary surfaces (formulas 6 and 7), a gray medium with mirror reflection at the boundary surfaces (formulas 33 and 34), or a selective medium with mirror reflection at the boundary surfaces (formulas 31, 32, and 37). These formulas are entirely correct. For a gray medium they can be used with any degree of accuracy; for a selective medium the accuracy of this solution is limited by our ignorance of the emissivities and the absorptivities of gases. The radiative heat transfer in a selective medium with isotropic reflection at the boundary surfaces can be estimated approximately from the amount of radiative heat transfer with mirror reflection, namely the latter amount multiplied by the respective ratio of heat transfer with isotropic reflection to heat transfer with mirror reflection for a gray medium.

The data on determining $\varepsilon(i, k)$ and $\varepsilon(i, i)$ as well as on their values can be found in [1, 4].

From these solutions for isotropic reflection and mirror reflection follow a few special cases: for a sphere with $\varphi_{ik} = 0$ and $\varphi_{ii} = 1{,}0$, for a stratum with $\varphi_{ik} = 1{,}0$ and $\varphi_{ii} = 0$, and for radiative heat transfer within a diathermal medium with $a_{ik} = a_{ii} = \varepsilon(i, k) = \varepsilon(i, i) = 0$. The sequence of the heat transfer calculations for radiation from a stratum has been shown in [1].

The formulas are valid also for radiative heat transfer within the space between two infinitely long coaxial cylinders.

The material presented here is also applicable to the calculation of selective radiation in various areas of heat engineering.

NOTATION

$q'_G(i)$	is the gas radiation energy absorbed by surface i, referred to radiant fluxes between surfaces i and k, per 1 m^2 of surface i area;
$q''_G(i)$	is the gas radiation energy absorbed by surface i, referred to radiant fluxes beyond surface k, per 1 m^2 of surface i area;
$q_G(k)$	is the gas radiation energy absorbed by surface k, per 1 m^2 of surface k area;
$q_k(i)$	is the energy emitted by surface k and absorbed by surface i, per 1 m^2 of surface k area;
$q_k(k)$	is the energy emitted by surface k and absorbed by surface k, per 1 m^2 of surface k area;
$q_i(k)$	is the energy emitted by surface i and absorbed by surface k per 1 m^2 of surface i area;
$q'_i(i)$	is the energy emitted by surface i and absorbed by surface i, referred to radiant fluxes between surfaces k and i, per 1 m^2 of surface i area;
$q''_i(i)$	is the energy emitted by surface i and absorbed by surface i, referred to radiant fluxes beyond surface k, per 1 m^2 of surface i area;
$q_c(i)$	is the intrinsic radiation of surface i per 1 m^2 area;
$q_c(k)$	is the intrinsic radiation of surface k per 1 m^2 area;
Q_{inc}	is the incident radiant flux;
Q_{eff}	is the effective radiant flux;
Q_R	is the resultant radiant flux;
F	is the surface area;
r	is the radius;
$d\omega$	is the differential of solid angle;
ω_0	is the solid angle subtending surface k as viewed from elements of surface i;
ϑ_k	is the angle between normal to surface k and the straight line joining two surface elements dF_i and dF_k;
ϑ_i	is the angle between normal to surface i and the straight line joining two surface elements dF_i and dF_k;
φ_{ik}	is the angular coefficient from surface i to surface k;
φ_{ii}	is the angular coefficient from surface i to itself;
$\omega = F_k/F_i = (r_k/r_i)^2$	
A	is the absorptivity of the surfaces;
R	is the reflectivity of the surfaces;
$a(x)$	is the absorptivity of the medium along path x;
ε_x, $\varepsilon(x)$	is the emissivity of the medium along path x;
a_{ik}	is the absorptivity of the medium between surfaces i and k;
a_{ii}	is the absorptivity of the medium between surface i and surface i;

$\varepsilon(i, k)$	is the emissivity of the medium between surfaces i and k;
$\varepsilon(i, i)$	is the emissivity of the medium between surface i and surface i;
d_i, D_x	are the respective transmittivities of the medium;
σ_0	is the black-radiation constant;
T	is the absolute temperature.

Subscripts

i	refers to surface i;
k	refers to surface k;
G	refers to gaseous medium;
R	refers to resultant flux;
eff	refers to effective flux;
inc	refers to incident flux.

LITERATURE CITED

1. A. S. Nevskii, Radiative Heat Transfer in Ovens and Furnaces [in Russian], Izd. Metallurgiya (1971).
2. H. C. Hottel and H. G. Mangelsdorf, Trans. AIChE, 31, 517-549 (1934-1935).
3. H. C. Hottel and R. B. Egbert, Trans. AIChE, 38, No. 3, 531-565 (1942).
4. A. S. Nevskii, "Thermal performance of metallurgical furnaces," Nauch. Trudy VNIIMT, No. 11, 145-165 (1965).